Temporal Homogeneity of Japanese Yen, Euro and Chinese Yuan Exchange Rate Behavior

Part I: Time Series Econometric Contrasts between Two Periods

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Abstract

Contrasting the individual and joint behavior of three daily exchange rates (the Japanese Yen, the Euro and the Chinese Yuan), all against a U.S. dollar, during the period of July 21, 2005 - July 31, 2008 with the corresponding behavior during the period of June 21, 2010 - December 30, 2016, this paper finds first that, while during both periods the daily rate of change in the Euro exchange rate obeys a white noise, for both periods and for both the Japanese Yen and the Chinese Yuan exchange rates their current daily rates of change depend on the previous daily rates of change 19 days in the past. Second, the temporal and cross-currency homogeneity thus observed for the two periods may not be mere coincidence and could be more than statistical in nature. Third, the unrestricted VAR modeling, whose lag length turns out two, detects for the former period as well as for the period of June 21, 2010 - August 10, 2015 no cointegration relationships among the three daily exchange rates; yet singling and separating out the Yuan's exchange rate just because of its inflexible nature appears inappropriate. For both periods, the VAR modeling of the three may still be meaningful for the managerial forecasting purposes.

1 Introduction

Kojima (2019) studies the individual and joint behavior of three daily exchange rates (the Japanese Yen, the Euro and the Chinese Yuan), all against a U.S. dollar, during the period ("V through 2016") of Monday,

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June 21, 2010 - Friday, December 30, 2016, the longest period of time when the Yuan was continuously less managed/controlled by China's central bank under (managed) flexible exchange rate system. Similar time series econometric study remains for the previous period ("III") of Thursday, July 21, 2005 - Thursday, July 31, 2008, the second longest period of time when China employed (managed) flexible exchange rate system.[1] Contrasting the findings between two periods, III and V through 2016, is thus a topic to be studied in the present paper, as further remarked below.

A priori (*deductively*) no theory postulates that exchange rates are cointegrated. Yet *a posteriori* (*inductively*) exploring for the evidence of a cointegration relationship is a worthwhile empirical/data-driven research. The two-fold purpose of the present paper is thus (as in Kojima 2019) to individually study the behavior of the three daily exchange rates in a univariate time series framework, and further to research the joint behavior of the three exchange rates by a multivariate time series model.

One particular question related to the purpose is how the two periods, III and V, compare with regard to the (univariate and multivariate) time series behavior. The question is asked since whether time series behavior of exchange rates including in particular the Chinese Yuan varies over time (that is, over differing periods) as well as with regard to currency is of empirical interest. If it does not, then one may infer temporal and cross-currency homogeneity of the exchange rate behavior, which would in turn lead to meaningful empirical (univariate and multivariate) time-series models of homogeneous exchange rate behavior, at least for the three exchange rates under study.

What is specifically meant in the present paper by temporal and cross-currency homogeneity of the exchange rate behavior is as follows: "Temporal homogeneity" means that an exchange rate has one or more parameters in common in its time series models for two or more nonoverlapping periods; "cross-currency homogeneity" means that a multiple exchange rates have one or more parameters in common in their time series models for a period; and "temporal and cross-currency homogeneity" means that a multiple exchange rates have one or more parameters in common in their time series models for two or more nonoverlapping periods. A negation of homogeneity as such is naturally behavioral heterogeneity.

[1]For exchange rate systems employed by China over differing periods such as Periods III and V, see, respectively, Panels 1 and 2 of Table 1.

Table 1 Exchange Rate Systems in China since 1994, together with Variability of Daily Rate of Change in CD (grCD):[a] Panel 1

I. Monday, January 3, 1994 - Tuesday, December 31, 1996 ($T = 767$ for Raw, Undifferenced Data):[b] *Flexible (Essentially, Pegged-to-U.S. Dollar) Exchange Rate System.*			
Statistics on Series grCD			
Observations 745[c] Skipped/Missing 21[d]			
Sample Mean	-0.000060	Variance	0.000002[e]
Minimum	-0.021362[f]	Maximum	0.020855[g]
Median	-0.000048		
II. 1997 - Wednesday, July 20, 2005: *Fixed Exchange Rate System.*			
III. Thursday, July 21, 2005 - Thursday, July 31, 2008 ($T = 761$ for Raw, Undifferenced Data):[h] *Managed Flexible Exchange Rate System.*			
Statistics on Series grCD			
Observations 760[i]			
Sample Mean	-0.000225	Variance	0.000001[j]
Minimum	-0.004599	Maximum	0.003163[k]
Median	-0.000169		
IV. August 2008 - Friday, June 18, 2010: *Fixed Exchange Rate System.*			

(Continued to Panel 2 of the Table)

[a]See Fig. 1. CD denotes CNYUSD, the Chinese Yuan exchange rate against a U.S. dollar. Source: BJidentify_fixdata_Jan31994-Dec311996output, BJidentify_fixdata_Jul212005-Jul312008output and BJidentify_fxdataoutput.

[b]This is the shaded period without vertical grid lines in Fig. 1. See Fig. 4 for dlogCD_t (= logged $\mathrm{CD}_t -$logged CD_{t-1}) that closely approximates grCD_t [$= (\mathrm{CD}_t\text{-}\mathrm{CD}_{t-1})/\mathrm{CD}_{t-1}$]. See Kojima (2019, Subsection 3.1) for the economic interpretation of logged series in first differences as a rate of change.

[c]This equals $T' - Missing = T - d - Missing = 767 - 1 - 21$ where T' denotes the effective sample size (the number of differenced data) and d the order of (consecutive) differencing required to compute the rate of change grCD; see Table 2 in Subsection 2.1 for the notation. Tuesday, January 4, 1994 - Tuesday, December 31, 1996.

[d]This is due to the dates when (raw) JD (denoting JPYUSD, the Japanese Yen exchange rate against a U.S. dollar) is available but neither CD nor ED (denoting EURUSD, the Euro exchange rate against a U.S. dollar): They are 11th, 36th, 65th, 66th, 106th, 131st, 231st, 266th, 291st, 361st and 387th dates; and thus daily rates of change are not available at twenty one dates (11th, 12th, 36th, 37th, 65th, 66th, 67th, 106th, 107th, 131st, 132nd, 231st, 232nd, 266th, 267th, 291st, 292nd, 361st, 362nd, 387th and 388th dates). For such details as exact dates see the very first output in the source list above and Appendix B.1.

[e]An unbiased sample variance (Doan 2007a, p.441). The (unbiased) sample standard deviation = 0.001382.

[f]Huge appreciation on Tuesday, December 20, 1994 (249th date).

[g]Huge devaluation on Monday, December 19, 1994 (248th date).

[h]This is the shaded period with vertical grid lines in Fig. 1.

[i]Friday, July 22, 2005 - Thursday, July 31, 2008.

[j]The (unbiased) sample standard deviation = 0.000956.

[k]Range (=Maximum-Minimum)=0.007762.

Panel 2

V. Monday, June 21, 2010 - Monday, August 10, 2015 ($T = 1286$ for Raw, Undifferenced Data):[a] *(Managed) Flixible Exchange Rate System.*			
Statistics on Series grCD			
Observations 1285[b] Skipped/Missing 1[c]			
Sample Mean	-0.000070	Variance	0.000001[d]
Minimum	-0.005912	Maximum	0.006042[e]
Median	-0.000075		
VI. Tuesday, September 1, 2015 - Friday, December 20, 2019/Present ($T = 1077$/More, for Raw, Undifferenced Data):[f] *(Managed) Flixible Exchange Rate System.*[g]			

[a]This is the shaded period without vertical grid lines in Fig. 1.

[b]Tuesday, June 22, 2010 - Monday, August 10, 2015.

[c]For why one missing see Appendix B.2.

[d]The (unbiased) sample standard deviation = 0.001139, which is larger than that for the period of Friday, July 22, 2005 - Thursday, July 31, 2008 above (see footnote j to Panel 1 of the table).

[e]Range =0.011954, which is larger than that for the period of Friday, July 22, 2005 - Thursday, July 31, 2008 in Period III (see footnote k to Panel 1 of the table).

[f]Drawn in Fig. 5, in particular for the period from early January 2018 to mid-May 2019, is the behavior of the daily Yuan falling and firming as most likely associated with the U.S.-China trade war during the period.

[g]The same system as for Period V.

1.1 Literature review

The past, fundamental literature includes Box and Jenkins (1976) and Kojima (1994, 2019), for univariate time series analysis and (multivariate) vector autoregressive (VAR) modeling. Focusing on Period V (through 2016), however, Kojima (2019) leaves out the behavior of the three exchange rates above for Period III (and Period I); possible temporal, as well as cross-currency, homogeneity of the exchange rate behavior over the varying periods is thus not yet investigated.

1.2 Data and the sample period

The three daily and monthly exchange rate data are all extracted from the Database Retrieval System (v2.11), available at the University of British Columbia's Sauder School of Business (http://fx.sauder.ubc.ca/data.html). Daily data are average daily rates and monthly data monthly averages to which the daily data are converted.[2] The sample period is

[2]See UBC Sauder's Website.

Period III (Thursday, July 21, 2005 - Thursday, July 31, 2008) [$T = 761$ Observations]. For the sample period see a note on the shaded period with vertical grid lines in Fig. 1.

Figure 1 Monthly Exchange Rates, January 1994 - December 2016 (Shaded: January 1994 - December 1996; July 2005 - July 2008 with vertical grid lines; June 2010 - July 2015). Note 1: Drawn for a clear exposition are EURUSD100(=ED×100) and CNYUSD10(=CD×10). Note 2: The shaded period with vertical grid lines is the third longest period of time when CD was continuously less managed/controlled by the central bank in China under (managed) flexible exchange rate system; this period corresponds to Period III as in Panel 1 of Table 1. (Incidentally, Period VI is the second longest period of such a time.)

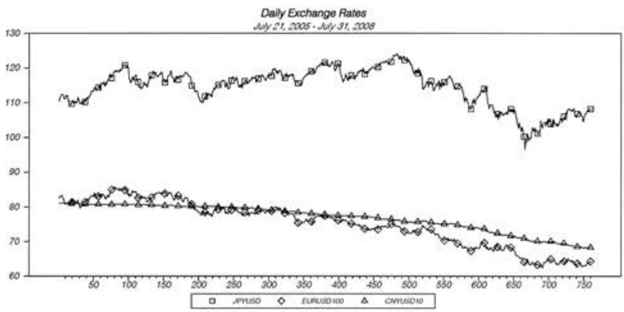

Figure 2 Daily Exchange Rates, Period III [T = 761 Observations]. Note: For Period III see Panel 1 of Table 1; see also Fig. 3.

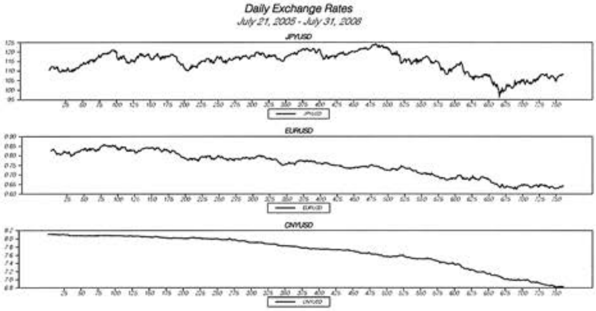

Figure 3 Daily Exchange Rates, Period III [$T = 761$ Observations].

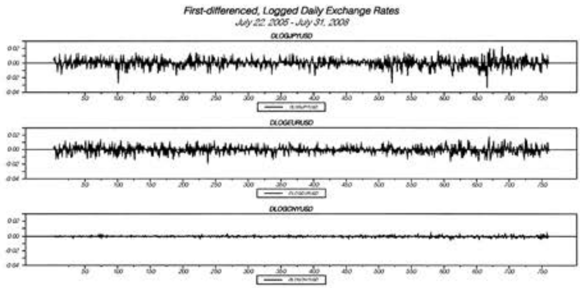

Figure 4 Logged Daily Exchange Rates in First Differences (Daily Rate of Change in Exchange Rates), Friday, July 22, 2005 - Thursday, July 31, 2008 (in Period III) [$1+d$ to T: 2 to 761, with $T'=T-d = 761-1$ where T' and d are as defined in Table 2 in Subsection 2.1].

The paper proceeds as follows: The relevant literature is reviewed in Section 1.1. Univariate time series models are identified and estimated in Section 2. Section 3 attempts to build VAR models to study the joint behavior of the daily exchange rates by computing roots of the companion matrix and conducting (F and chi-squared) tests on three differing nulls of lagged regressor(s) being excluded/omitted. Several concluding remarks on the contrast between the two periods III and V are made in the context of temporal and cross-currency homogeneity, in the final section. Two appendices follow: Figure appendix and table appendices.

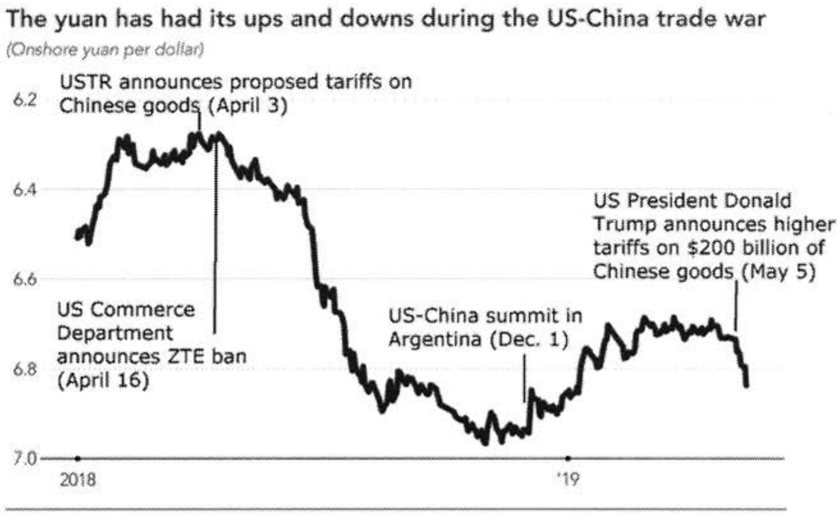

Figure 5 The Daily Yuan Behavior, early January 2018 - mid-May 2019, during the U.S.-China Trade War (in Period VI). Note: The weaker/softer Yuan is drawn in the downward direction (unlike in Figs. 1 - 3); the data are readily retrieved from the database as described in Subsection 1.2; an investigation remains to be done in the future for the figure. Source: Cho (2019).

2 Univariate Modeling: Identification and Estimation

With X_t and a_t denoting, respectively, the raw data and the white-noise error term, and the usual notation, the univariate, multiplicative seasonal autoregressive integrated moving average model, SARIMA $(p, d, q; P, D, s, Q)$, for $X_t^\ell (= \log X_t)$ is written as

$$\phi(B)\Phi(B^s)(1-B)^d(1-B^s)^D X_t^\ell = \theta(B)\Theta(B^s)a_t \qquad (1)$$

where $\phi(B), \Phi(B^s), \theta(B)$ and $\Theta(B^s)$ are, respectively, AR, SAR, MA and SMA multinomials of backshift operator B, which, with $\phi_0 = \Phi_0 =$

$\theta_0 = \Theta_0 = -1$, are written as:

$$\begin{aligned} \phi(B) &= -\sum_{i=0}^{p} \phi_i B^i;\ \Phi(B^s) = -\sum_{i=0}^{P} \Phi_i B^{is}; \\ \theta(B) &= -\sum_{i=0}^{q} \theta_i B^i;\ \Theta(B^s) = -\sum_{i=0}^{Q} \Theta_i B^{is}. \end{aligned} \tag{2}$$

Also:

$$W_t^\ell = (1-B)^d(1-B^s)^D X_t^\ell. \tag{3}$$

(For further details see Kojima 2019, Subsection 3.1.)

Estimated models for Period III here will be contrasted with those estimated by Kojima (2019) for Period V through 2016.[3]

2.1 Identification

Univariate time series models for exchange rates are identified as summarized in Kojima (2019, Subsection 3.1). The following table is quoted from there and will be subsequently referred to:

Table 2 Time Framework for Raw (Undifferenced) Data, Differenced Data and Residuals Series

Raw (Undifferenced) Data X_t	Differenced, Logged Data W_t^ℓ	Residuals Series e_t[a]
1		
2		
⋮		
$1+d+sD$	1	
⋮	⋮	
$1+d+sD+\underline{\max\{p,sP\}}$	$1+\underline{\max\{p,sP\}}$	1
⋮	⋮	⋮
T	$T'(=T-d-sD)$	$T'^r(=T'-\underline{\max\{p,sP\}})$

[a]For this notation see Kojima (2019, Subsection 5.3.3).

Based on Fig. 10 (drawing SACF[4] and SPACF[5]) in Appendix A, the

[3]Period V and the period of Tuesday, August 11, 2015 - Friday, December 30, 2016 combined.

[4]Sample autocorrelation function.

[5]Sample partial autocorrelation function.

logged daily JD in first differences (the daily rate of change in JD)[6] is identified as an AR[4, 20],[7] to be estimated in the following subsection.[8]

Based on Fig. 14 in Appendix A, the logged daily ED in first differences (the daily rate of change in ED) is appropriately identified as a white noise, to be estimated in the following subsection.

Based on Fig. 17 in Appendix A, the logged daily CD in first differences (the daily rate of change in CD) is identified as AR[19], an AR model only with ϕ_{19}, to be estimated in the following subsection.[9]

2.2 Estimation

Univariate time series models for exchange rates are estimated following Kojima (2019, Subsection 3.2).

2.2.1 Logged daily JD in first differences (Daily rate of change in JD)

First, AR[4, 20] with a constant is estimated to find the constant statistically insignificant at any conventional levels: See Table 3 just below along with Fig. 21 in Appendix A. AR[4, 13, 19, 20] without a constant is then estimated: See Table 4 just below and Fig. 22 in Appendix A.

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[6]For this interpretation see footnote *b* to Panel 1 of Table 1.

[7]A pair of square brackets means that ϕ_4 and ϕ_{20} are *only* included in the AR model, a time series model for differenced, logged series W_t^{ℓ} as computed by Eq. (3).

[8]Notice that an MA[4, 20] model may be equally identified based on SACF (middle) in Fig. 10. An AR model identified based on SPACF is preferred in the present paper, for it can be more conveniently interpreted in the context of a Markov process, i.e., AR(1), versus a non-Markov process (Nelson 1973, pp.38-39), as will be seen in Subsection 2.2.1. Even with the AR model, Fig. 21 in Appendix A for the estimation phase later shows that the residuals autocorrelations (Residuals SACF) at lags 4 and 20 will not be statistically significant (implying no MA term to be added at either lag) as desired.

[9]Notice that an MA model with θ at lag 19 may be equally identified based on SACF (middle) in Fig. 17. An AR model is again preferred, for it can be more conveniently interpreted in the context of a Markov process versus a non-Markov process, as will be seen in Subsection 2.2.3. Even with the AR model, Fig. 24 in Appendix A for the estimation phase later shows that the Residuals SACF at lag 19 will not be statistically significant (suggesting no MA term to be added at the lag) as desired.

Table 3 Estimated AR[4, 20] Model for Logged Daily JD in First Differences (Daily Rate of Change in JD):[a] Period III; $T = 761^b$

Box-Jenkins - Estimation by LS Gauss-Newton[c]				
Convergence in 6 Iterations. Final criterion was $0.0000000 \leq 0.00001$				
Dependent Variable TRANSFRM[d]				
Usable Observations	740[e]	DF[f]	737[g]	
Centered R**2	0.986	R Bar **2	0.986	
Uncentered R**2	1.000	T x R**2	739.999	
Mean of Dependent Variable		4.739		
Std Error of Dependent Variable[h]		0.050		
Standard Error of Estimate[i]		0.006		
Sum of Squared Residuals		0.026		
Regression F(2,737)[j]		25623.882		
Significance Level of F		0.000		
Log Likelihood		2743.204		
Durbin-Watson Statistic		2.020		
Q(36-2)		35.641		
Significance Level of Q		0.391		
Variable	Coeff	Std Error	T-Stat	Signif
1. CONSTANT	-0.000	0.000	-0.173	0.862
2. AR{4}[k]	-0.096	0.036	-2.630	0.009
3. AR{20}	-0.095	0.037	-2.602	0.009

[a]W_t^ℓ as computed by Eq. (3) with $d = 1$ and s and $D = 0$: This applies to all the remaining tables in Subsection 2.2 and Section 4.

[b]Source: BJestimate_outputJPYUSD1.

[c]See Doan (2007b, pp.176-179) for the detailed description of the output in the table: In particular, the (marginal) siginificance level (of the F-statistic, the Q-statistic and the t-statistic) is called the P-value.

[d]"Dependent Variable TRANSFRM" is a *yet undifferenced*, logged exchange rate in levels, denoted by X_t^ℓ in Eq. (1) or (3): See Fig. 10 in Appendix A.

[e]"Usable Observations" here is set equal to the number of residuals, T'^r, which equals $\underline{T' - \max\{p, sP\}} = T - d - \underline{sD - \max\{p, sP\}} = 761 - 1 - \underline{20}$ (see the footnote immediately below Eq. (3) in Kojima 2019, Subsection 3.1). See Table 2, too.

[f]Degrees of Freedom.

[g]This is equal to "Usable Observations" (i.e., T'^r)−the number of parameters excluded for a block F test (which are the constant, ϕ_4 and ϕ_{20}) =740-3. (For a block F test see Panel 1 of Table 10.)

[h]This is an unbiased standard deviation of the dependent variable X_t^ℓ (with the divisor being $T - 1$).

[i]This is $\sqrt{\text{“Sum of Squared Residuals” (just below)/“DF” (just above)}}$.

[j]"RATS only does the F-test for ordinary least squares regression with a constant, since it is meaningless in most other situations." "The F-test statisitc tests the null that all coefficients in the regression (other than the intercept) are zero." See Doan (2007b, pages 176 and 178).

[k]This denotes W_{t-4}^ℓ, in Eqs. (1) - (3), associated with ϕ_4, an AR parameter at lag 4.

Table 4 Estimated AR[4, 13, 19, 20] Model for Logged Daily JD in First Differences (Daily Rate of Change in JD): Period III; $T = 761^a$

Box-Jenkins - Estimation by LS Gauss-Newton				
Convergence in 2 Iterations. Final criterion was 0.0000000 ≤ 0.00001				
Dependent Variable TRANSFRM				
Usable Observations	740	DF	736^b	
Centered R**2	0.986	R Bar **2	0.986	
Uncentered R**2	1.000	T x R**2	739.999	
Mean of Dependent Variable		4.739		
Std Error of Dependent Variable		0.050		
Standard Error of Estimate		0.006		
Sum of Squared Residuals		0.026		
Log Likelihood		2747.744		
Durbin-Watson Statistic		2.013		
Q(36-4)		27.2246		
Significance Level of Q		0.707		
Variable	Coeff	Std Error	T-Stat	Signif
1. AR{4}	-0.097	0.036	-2.670	0.008
2. AR{13}	0.083	0.036	2.269	0.024
3. AR{19}	0.072	0.036	1.983	0.048
4. AR{20}	-0.092	0.036	-2.540	0.011

[a]Source: BJestimate_outputJPYUSD2.

[b]This is equal to "Usable Observations" (i.e., T'^r)−the number of parameters excluded for a block F test (which are $\phi_4, \phi_{13}, \phi_{19}$ and ϕ_{20}) =740-4.

Contrasting Table 4 with Kojima's (2019) Table 4 for Period V through 2016 AR[4, 13, 19, 20] model for Period III (as shown in Table 4 just above) is now contrasted with AR[19] for Period V through 2016 (as shown in Kojima's (2019) Table 4 for Period V through 2016, which is quoted as Table 5 just below). Notice that while the AR parameter ϕ_{19} is common to models for both Period III and Period V through 2016, additional AR parameters are included for the former period. The results suggest that (univariate) JD behaves in a more complex manner during the former period than during the latter period.

A *statistical* reason for including AR parameter ϕ_{19} in particular for two periods may be summarized as follows:

[Period III] Recall from Vandaele (Japanese Edition, 1988, p.35) (and Kojima 1994, p.28 refering to Nelson 1973, p.115 and Hokstad 1983, p.177) that for a univariate time series model, its CCF_l[10] between the

[10]Cross-correlation function at lag l.

Table 5 [Kojima's (2019) Table 4] Estimated AR[19] Model for Logged Daily JD in First Differences (Daily Rate of Change in JD): Period V through 2016; $T = 1634^a$

Box-Jenkins - Estimation by LS Gauss-Newton				
Convergence in 2 Iterations. Final criterion was 0.0000000 $\leq$ 0.00001				
Dependent Variable TRANSFRM				
Usable Observations	1614^b	DF	1613^c	
Centered R**2	0.999	R Bar **2	0.998	
Uncentered R**2	1.0	T x R**2	1613.997	
Mean of Dependent Variable		4.569		
Std Error of Dependent Variable		0.160		
Standard Error of Estimate		0.006		
Sum of Squared Residuals		0.062		
Log Likelihood		5911.146		
Durbin-Watson Statistic		1.977		
Q(36-1)		23.756		
Significance Level of Q		0.925		
Variable	Coeff	Std Error	T-Stat	Signif
1. AR{19}	0.064	0.025	2.551	0.011

[a]Source: BJetimate_jpyModelBoutput.

[b]"Usable Observations" here is set equal to the number of residuals, T'^r, which equals $T' - \max\{p, sP\} = T - d - sD - \max\{p, sP\} = 1634 - 1 - 19$ (see the footnote immediately below Eq. (3) in Kojima 2019, Subsection 3.1). See Table 2, too .

[c]This is equal to "Usable Observations" (i.e., T'^r)−the number of parameters excluded (except for the constant) for a block F test, which is ϕ_{19}=1614-1. (For a block F test see Panel 1 of Table 10.)

white-noise error term a_{t-l} and *past* differenced, logged data W_t^ℓ at lag $l \leq -1$ is zero under the assumed independence between the two such series.[11] A large, nonzero SCCF_l[12] at lag $l < 0$ thus suggests an AR parameter to be inserted at that $|l|$, as proposed by Hokstad(1983) for diagnostic checking of estimated models. Adding AR parameter ϕ_{19} then leads, as desired, to zero SCCF_{-19} between residuals $e_{t-(-19)}$ and past differenced data W_t^ℓ (at lag $l = -19$), as readily seen by contrasting Figs. 21 and 22 in Appendix A.

[Period V through 2016] AR parameter ϕ_{19} needs to be added due

[11]For an AR (1) model $W_t^\ell = \phi W_{t-1}^\ell + a_t$, for example, the current white-noise error term a_t and past differenced data W_{t-1}^ℓ are assumed independent. Meanwhile, a_t and *present and future* data $W_t^\ell, W_{t+1}^\ell, \ldots$ are assumed dependent and thus CCF_l between a_{t-l} and data W_t^ℓ at lag $l \geq 0$ may be *nonzero*.

[12]Sample CCF at lag l.

to the nonzero SPACF_l of the differenced, logged data W_{t-l}^{ℓ} for $l = 19$ detected at the initial identification stage (see Kojima 2019, Subsection 3.1). Thus, what is inferred for Period III above applies to Period V through 2016 as well (for which diagnostic checking suggests no additional parameters to be inserted, as shown by Kojima 2019, Subsection 3.2.1).

[Both periods] (First-differenced, logged daily) JD (Daily rate of change in JD), W_t^{ℓ} as computed by Eq. (3), thus obeys an AR model that is a non-Markov process.

2.2.2 Logged daily ED in first differences (Daily rate of change in ED)

First, a white noise model with a constant is estimated to find the constant statistically insignificant at any conventional levels. The constant is thus excluded: See Table 6 just below and Fig. 23 in Appendix A.

Table 6 Estimated White Noise Model for Logged Daily ED in First Differences (Daily Rate of Change in ED): Period III; $T = 761^a$

Box-Jenkins - Estimation by LS Gauss-Newton			
Dependent Variable TRANSFRM[b]			
Usable Observations	760[c]	DF	760
Centered R**2	0.997	R Bar **2	0.997
Uncentered R**2	1.0	T x R**2	759.804
Mean of Dependent Variable		-0.290[d]	
Std Error of Dependent Variable		0.089	
Standard Error of Estimate		0.005	
Sum of Squared Residuals		0.018	
Log Likelihood		2969.982	
Durbin-Watson Statistic		2.013	
Q(36-0)		44.216	
Significance Level of Q		0.163	
NO ESTIMATED COEFFICIENTS			

[a]Source: BJestimate_outputEURUSD.

[b]"Dependent Variable TRANSFRM" is a *yet undifferenced*, logged exchange rate in levels, denoted by X_t^{ℓ} in Eq. (1) or (3): See Fig. 14 in Appendix A.

[c]This equals $T'^{r} = T' \underline{- \max\{p, sP\}} = T - d \underline{- 0} = 761 - 1$. See Table 2.

[d]The mean is negative because the raw (i.e., yet unlogged) ED is (positive but) less than 1: See the middle line graph in Fig. 3.

Contrasting Table 6 with Kojima's (2019) Table 6 for Period V through 2016 As shown in Kojima's (2019) Table 6 (which is not quoted here), a white noise model is, too, found appropriate for the logged daily ED in first differences (daily rate of change in ED), W_t^ℓ as computed by Eq. (3), for Period V through 2016; in other words, logged daily ED in levels, X_t^ℓ, obeys a random walk model (a Markov process) in Period III as well.

2.2.3 Logged daily CD in first differences (Daily rate of change in CD)

See Table 7 just below and Fig. 24 in Appendix A.

Table 7 Estimated AR[19] Model for Logged Daily CD in First Differences (Daily Rate of Change in CD): Period III; $T = 761^a$

Box-Jenkins - Estimation by LS Gauss-Newton				
Convergence in 2 Iterations. Final criterion was $0.0000000 \leq 0.00001$				
Dependent Variable TRANSFRM[b]				
Usable Observations	741[c]	DF	740[d]	
Centered R**2	1.0	R Bar **2	1.0.	
Uncentered R**2	1.000	T x R**2	741.000	
Mean of Dependent Variable		2.036		
Std Error of Dependent Variable		0.051		
Standard Error of Estimate		0.001		
Sum of Squared Residuals		0.001		
Log Likelihood		4079.646		
Durbin-Watson Statistic		2.027		
Q(36-1)		43.833		
Significance Level of Q		0.145		
Variable	Coeff	Std Error	T-Stat	Signif
1. AR{19}	0.139	0.038	3.672	0.000

[a]Source: BJestimate_outputCNYUSD.

[b]"Dependent Variable" here is a *yet undifferenced*, logged exchange rate in levels, denoted by X_t^ℓ in Eq. (1) or (3): See Fig. 17 in Appendix A.

[c]"Usable Observations" here is set equal to $T' - \max\{p, sP\} = T - d - sD - \max\{p, sP\} = 761 - 1 - 19$.

[d]This is equal to "Usable Observations"-the number of parameters excluded for a block F test (which is ϕ_{19}) =741-1.

Contrasting Table 7 with Kojima's (2019) Table 10 for Period V through 2016 The logged daily CD in first differences (daily rate of change in CD) for Period V through 2016, which is found to contain two permanent (level) shifts on August 11 and 12 in 2015, requires an intervention model as shown in Kojima's (2019, Section 4) (Table 10 in particular there, which is quoted as Table 8 just below where footnotes d and e are newly added for the present paper). Yet the AR parameter ϕ_{19} is common to models for both Period III and Period V through 2016, although only marginally significant for the latter period.

A *statistical* reason for including AR parameter ϕ_{19} in particular for two periods is the same as for JD:

[Period III] AR parameter ϕ_{19} needs to be added due to the nonzero SPACF_l of the differenced, logged data W_{t-l}^{ℓ} for $l = 19$ detected at the initial identification stage (see Subsection 2.1). Thus, what is inferred for Period V through 2016 below applies to Period III as well.

[Period V through 2016] Adding AR parameter ϕ_{19} leads, as desired, to zero SCCF_{-19} between residuals $e_{t-(-19)}$ and past differenced data W_t^{ℓ} (at lag $l = -19$), as readily seen in Kojima (2019, Subsection 4.3.2).

[Both periods] (First-differenced, logged daily) CD (Daily rate of change in CD), W_t^{ℓ} as computed by Eq. (3), too, obeys a time-series model that is a non-Markov process.

2.2.4 Economic implications for temporal homogeneity

Whereas *[a.Markov process]* in both periods current (logged) daily EUR /USD in levels (X_t^{ℓ}) depends only on just preceding data (X_{t-1}^{ℓ}), *[b.non-Markov process]* for both periods and for *both* JD and CD current daily rate of change in data (W_t^{ℓ}) depends on previous daily rate of change in data 19 days in the past (W_{t-19}^{ℓ}). The latter *[b]* is observed even while in both two periods Japan employs a flexible exchange rate system and China a managed flexible exchange rate system as seen in Table 1.

The temporal and cross-currency homogeneity observed for the two periods (even with the intervention model for CD for Period V through 2016) may not be mere coincidence as to time and currency and could be more than statistical in nature as documented in the preceding subsections 2.2.1 through 2.2.3. What is essentially behind the economic implications such as *[a]* versus *[b]* (including the statistically significant ϕ_{19} in common) is not readily evident, however. Further investigation is needed in qualitative business and economic dimensions, which is beyond the scope of the paper.

Yet two quick, statistical preliminaries to the further study of *[a]* ver-

Table 8 [Kojima's (2019) Table 10] [Third, Final Version] Estimated Intervention Model for Logged Daily CD in First Differences (Daily Rate of Change in CD): Period V through 2016; $T = 1634^a$

Box-Jenkins - Estimation by LS Gauss-Newton				
Convergence in 7 Iterations. Final criterion was $0.0000036 \leq 0.00001$				
Dependent Variable TRANSFRM				
Usable Observations	1614[b]	DF	1610	
Centered R**2	0.998	R Bar **2	0.998	
Uncentered R**2	1.0	T x R**2	1613.999	
Mean of Dependent Variable		1.850		
Std Error of Dependent Variable		0.032		
Standard Error of Estimate		0.001		
Sum of Squared Residuals		0.003		
Log Likelihood		8418.768		
Durbin-Watson Statistic		2.013		
Q(36-2)		45.604		
Significance Level of Q		0.0882		
Variable	Coeff	Std Error	T-Stat	Signif
1. AR{19}	0.040	0.025	1.614	0.107
2. MA{5}	0.073	0.025	2.943	0.003
3. N_PSAUG112015{0}	0.019	0.001	14.082	0.000
4. N_PSAUG122015{0}	0.010	0.001	7.504	0.000
Statistics on Series RESIDS				
Observations	1614[c]			
Sample Mean	-0.000001	Variance	0.000002	
Standard Error[d]	0.001314	of Sample Mean[e]	0.000033	
t-Statistic (Mean=0)	-0.024444	Signif Level	0.980501	
Skewness	0.154087	Signif Level (Sk=0)	0.011574	
Kurtosis (excess)	5.879190	Signif Level (Ku=0)	0.000000	
Jarque-Bera	2330.874394	Signif Level (JB=0)	0.000000	

[a]Source: BJetimate_cnyModelB-ARMA_IntrvModeloutput.

[b]This equals $T'^r = T' - \max\{p, sP\} = T - d - p = 1634 - 1 - 19$. See Table 2 and the footnote to Table 4 in Kojima (2019, Subsection 3.2.1). For the model (19) in Kojima (2019, Subsection 4.3.1), $T' = T - 1$ and $T'^r = T' - \max\{p, sP\}$; thus the differenced data start at $1 + d + sD = 1 + 1 = 2$ and the residuals at $1 + d + sD + \max\{p, sP\} = 1 + 1 + 19 = 21$.

[c]See the footnote immediately above.

[d]*[Newly added for the present paper]* This is the unbiased standard deviation of residuals, computed usually as $\sqrt{\text{(unbiased) “Variance” (just above)}}$ (Doan 2007a, p.441).

[e]*[Newly added for the present paper]* This is the standard error of sample mean, computed usually as "(unbiased) standard deviation ("Standard Error")/$\sqrt{\text{“Observations” (just above)}}$ (Doan 2007a, p.441).

sus *[b]* are in order:[13]

A statistical preliminary on a daily basis One preliminary related to *[a]* versus *[b]* in Period III may be to look at SACF and SPACF of the *second*-order differences of logged daily JD and CD for their stationarity,[14] which are drawn, respectively, in Figs. 11 and 18 in Appendix A. Clearly the second-order (and even third-order) differencing will not lead to stationarity, implying that a white noise model will not apply to either of logged JD and CD in second differences. (In effect, neither a random walk model nor a Markov process will apply to either of logged JD and CD in levels or first differences.)

A statistical preliminary on a monthly basis With statistically significant ϕ_{19} in the *daily* data models for Period III it would be useful to interpret "*19 trading days plus four weekends (8 days)*" as "*nearly one month long.*" This interpretation might make it possible to infer that logged *monthly* exchange rates in levels may obey a random walk or a Markov process. Another preliminary may then be to look at SACF and SPACF of the first-order differences of logged monthly JD and CD, drawn, respectively, in Figs. 12 and 19 in Appendix A. Indeed, the first-order differences of logged monthly JD are a white noise, as inferred, whereas those of CD are not.[15]

Further, Figs. 13 and 20 in Appendix A, which draw SACF and SPACF of the first-order differences, respectively, of logged monthly JD and CD during Period V through 2016 for which the daily time series models, too, contain ϕ_{19},[16] show that the first-order differences of neither logged monthly JD nor CD obey a white noise model,[17] unlike those

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[13] Usual univariate testing for a unit root (as in Kojima 2019, Subsection 5.2.1) is not conducted, but rather studied below are SACF and SPACF of the data.

[14] For the second-order differencing see the first footnote to Subsection 3.2.1.

[15] The latter rather appear to be identified as AR(2) or ARMA(2,2), neither of which is a Markov process. (Incidentally, as shown in Fig. 15, logged monthly ED in first differences do not obey a white noise model or a Markov process in Period III, either.)

[16] See, respectively, Tables 5 and 8.

[17] The former and the latter rather appear to be identified, respectively, as AR[1,15] or ARMA(1,2) and as AR(1) or ARMA(1,1), of which AR(1) is the only Markov process. (Meanwhile, however, as shown in Fig. 16, logged monthly ED in first differences does obey a white noise model during Period V through 2016 when the daily counterpart, too, obeys a white noise model as shown in Subsection 2.2.2. This may imply that the statistically significant ϕ_{19} in a first-differenced, logged daily

of JD in Period III. The preliminary here on a monthly basis, thus, apparently fails to clarify what is consistently (over time, that is, over differing periods) behind the statistically significant ϕ_{19} in the daily time series models for JD and CD.

3 VAR Modeling

We now turn to VAR modeling for Period III, which will be contrasted with that by Kojima (2019) for Period V.[18] Recall in particular that the period beyond Monday, August 10, 2015 is not considered in the present section, for the reason explained at the beginning of Kojima (2019, Section 5).

There are two types of VAR that may be studied: The cointegrated VAR (or VECM[19]) and the unrestricted VAR. As argued in Kojima (2019, Subsection 5.1), "If, *a priori* (*deductively*), there are equilibrium conditions to be satisfied by the three daily exchange rates, JD, ED and CD, then the cointegrated VAR (or VECM) is the one to be used to test for the equilibrium conditions (or the cointegration relations).

If there are no such conditions or restrictions a priori, then the unrestricted VAR may be more appropriate. The present study presents no equilibrium condition a priori and thus will rely on the unrestricted VAR. ..."

Yet, relying on the cointegrated VAR Kojima (2019) found for Period V that *a posteriori* the VAR modeling detects no cointegration relationships among the three daily exchange rates, JD, ED and CD. For Period III, thus, the present paper omits entirely the cointegrated VAR but rather employs an alternative (additional) method of detecting cointegration relations, based on roots (eigenvalues) of the companion matrix; focused on thus is the *unrestricted* VAR alone.

3.1 A preliminary: Histograms and scatter diagrams

As a preliminary to the VAR modeling, histograms and (bivariate) scatter diagrams for Period III (as drawn in Figs. 6 and 7 just below) are

model is not a neccesity for a first-differenced, logged monthly model to be white noise. Further study remains.)

[18]For Period V (with $T = 1286$) for VAR modeling in the present section, see Panel 2 of Table 1 and the longest shaded period in Fig. 1 drawing the corresponding three monthly exchange rates.

[19]An abbreviation of a vector error-correction model.

contrasted with those for Period V (as drawn in Kojima's (2019) Figs. 21 and 22 for Period V, which are quoted, respectively, as Figs. 8 and 9 just below).[20]

For Period III Fig. 6 shows that, with a possibility of spurious correlations, the contemporaneous relations in levels are positive between any pair of the three exchange rates. Excluding such spuriosity, Fig. 7 evidences no contemporaneous relations (in rates of change) either between JD and CD or between ED and CD, implying that, a posteriori, no equilibrium condition or cointegration relation appears to be detected; meanwhile the figure appears to evidence a contemporaneous positive, though quite weak, relation (in rates of change) between JD and ED. To confirm this a posteriori finding, it would be useful to conduct a cointegration test (such as the one conducted for Period V in Kojima 2019, Subsection 5.2).

For Period III the present paper will not conduct such a test but rather employ an alternative method, as explained in the following subsection.

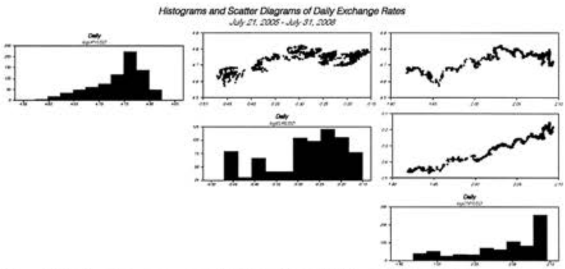

Figure 6 Histograms and Scatter Diagrams of Logged Daily Exchange Rates, Period III.

[20]Kojima (2019, Subsection 5.1) infers for Period V that "Figs. 21 and 22 ... would be useful to *a posteriori* (*inductively*) look at any possibility of correlations among the three daily exchange rates (respectively, logged and first-differenced logged ones) during the sample period V (Monday, June 21, 2010 - Monday, August 10, 2015). Fig. 21 may show that, with a possibility of spurious correlations, the contemporaneous relations in levels are positive between JD and ED, whereas negative between JD and CD and between ED and CD. Excluding such spuriosity, Fig. 22 evidences no contemporaneous relations (in rates of change) between any pair of the three exchange rates, implying that, a posteriori, no equilibrium condition or cointegration relation appears to be detected."

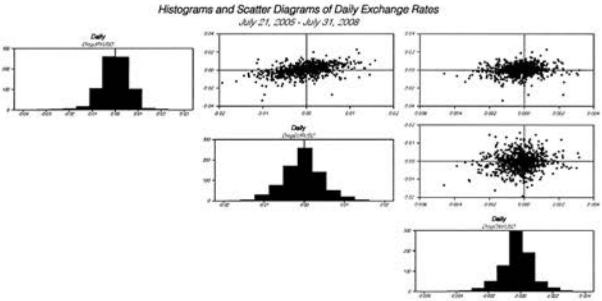

Figure 7 Histograms and Scatter Diagrams of First Differences of Logged Daily Exchange Rates (Daily Rates of Change in Exchange Rates), Friday, July 22, 2005 - Thursday, July 31, 2008 (in Period III). Note: Logged exchange rates in first differences (daily rates of change in exchange rates) for the period here are drawn in Fig. 4.

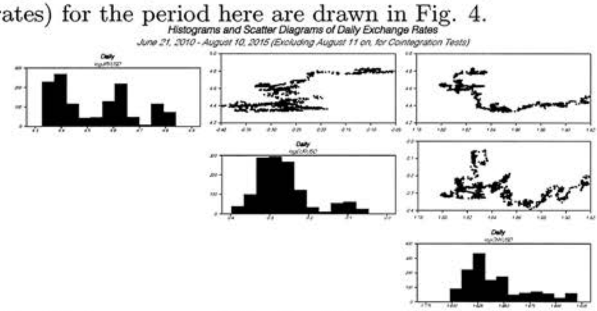

Figure 8 Histograms and Scatter Diagrams of Logged Daily Exchange Rates, Period V.

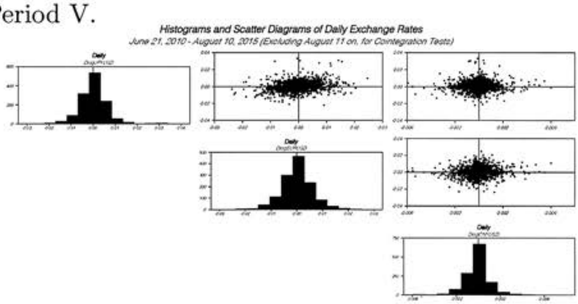

Figure 9 Histograms and Scatter Diagrams of First Differences of Logged Daily Exchange Rates (Daily Rates of Change in Exchange Rates), Tuesday, June 22, 2010 - Monday, August 10, 2015 (in Period V). Note: For the plot of first-differenced, logged exchange rates in first differences for the period here, see Kojima (2019, Fig. 7).

3.2 Unrestricted VAR modeling

The present subsection will build unrestricted VAR models, to study whether or not each lagged exchange rate is still to be included in the (entire) VAR model even with no a priori equilibrium condition or cointegration relation being detected, thereby exploring for the possibility of the three exchange rates behaving jointly during Period III, when, just as during Period V, the Chinese Yuan was continuously less managed/controlled by the central bank in China under (managed) flexible exchange rate system (see Section 1 referring to Table 1).

Denoting the nth-order column vector and a lag length, respectively, by $\boldsymbol{y}_t$ and L, we consider the VAR(L) model including a constant $\boldsymbol{\mu}$ but without the term $\boldsymbol{\Psi D}_t$ (centered seasonal dummies):[21]

$$\boldsymbol{y}_t = \sum_{l=1}^{L} \boldsymbol{\Phi}_l \boldsymbol{y}_{t-l} + \boldsymbol{\mu} + \boldsymbol{a}_t. \tag{4}$$

The test results in Table 12 in Appendix B.3, combined together, show that the appropriate lag length L for the daily exchange-rate VAR model (4) is as short as 2 (days). Note that this is exactly the same as for VAR modeling for Period V: See Kojima (2019, Subsection 5.3.1).

The unrestricted VAR model to be studied is thus Eq. (4) with $n = 3$ (that is, three logged daily exchange rates, $\log\mathrm{JD}_t$, $\log\mathrm{ED}_t$ and $\log\mathrm{CD}_t$)[22] and $L = 2$:[23]

$$\boldsymbol{y}_t = \sum_{l=1}^{2} \boldsymbol{\Phi}_l \boldsymbol{y}_{t-l} + \boldsymbol{\mu} + \boldsymbol{a}_t. \tag{5}$$

The estimated VAR(2) model for Eq. (5) may be written as:[24]

$$\boldsymbol{y}_t = \sum_{l=1}^{2} \hat{\boldsymbol{\Phi}}_l \boldsymbol{y}_{t-l} + \hat{\boldsymbol{\mu}} + \boldsymbol{e}_t \tag{6}$$

where: $\hat{\boldsymbol{\Phi}}_l, \hat{\boldsymbol{\mu}}$ and $\boldsymbol{e}_t$ denote, respectively, estimates of $\boldsymbol{\Phi}_l, \boldsymbol{\mu}$ and $\boldsymbol{a}_t$; in particular, $\boldsymbol{e}_t$ is the residuals vector.

---

[21] See Kojima (2019, Eq. (29), being augmented with the term $\boldsymbol{\Psi D}_t$, in Subsection 5.2.2).

[22] These are each denoted, in univariate time series models, by "Dependent Variable TRANSFRM" (or X_t^{ℓ}) in Tables 3 - 8 in Subsection 2.2.

[23] See Kojima (2019, Eq. (32) in Subsection 5.3.2).

[24] See Kojima (2019, Eq. (33) in Subsection 5.3.3).

3.2.1 An alternative, additional method of detecting cointegration relations, based on roots (eigenvalues) of the companion matrix

The companion matrix is the idea proposed by Juselius(1994) (published in 1995; see also Juselius 2006, pp.50-52).[25] Table 9 below is constructed based on the companion matrix $\hat{\mathbf{\Phi}}$ for the estimated VAR(2) model (6):[26]

$$\hat{\mathbf{\Phi}} = \begin{bmatrix} \hat{\mathbf{\Phi}}_1 & \hat{\mathbf{\Phi}}_2 \\ \boldsymbol{I}_3 & \mathbf{0} \end{bmatrix}$$

where: $\boldsymbol{I}_3$ is the third-order indentity matrix;

$$\hat{\mathbf{\Phi}}_1 = \begin{bmatrix} 0.957 & 0.058 & -0.006 \\ 0.004 & 0.977 & 0.024 \\ -0.002 & 0.066 & 0.877 \end{bmatrix} \text{for first lag;}$$

$$\hat{\mathbf{\Phi}}_2 = \begin{bmatrix} 0.029 & -0.061 & 0.023 \\ -0.009 & 0.004 & 0.011 \\ 0.001 & -0.064 & 0.122 \end{bmatrix} \text{for second lag}$$

---

[25] See Harris (1995, p.89), with italic phrases with parentheses below being added by the author of the present paper:

"However, it is also important to use any additional information that can support the choice of r. ... Thus the eigenvalues (i.e., roots) of ... the companion matrix are considered since these provide additional confirmation of how may $(n - r)$ roots are on the unit circle and thus the number of r cointegration relations. The matrix is defined by ... There are ten roots of the companion matrix in the present example, since $n \times k = 10$ *(where: n =the number of potentially endogenous variables, k =the number of lags in AR; in the present example, $n = 5, k = 2$).* The moduli of the 3 largest roots are 0.979, 0.918 and 0.918, ... indicating all roots are inside the unit circle, with the three largest close to unity. This suggests that $n - r = 5 - r = 3$, and thus there are (r =)two cointegration relations. The fact that all roots are inside the unit circle is consistent with the endogenous variables comprising I(1) processes, although it is certainly possible that the largest root is not significantly different from 1. If any of the roots are on or outside the unit circle, this would tend to indicated an I(2) model, requiring second-order differencing to achieve stationarity. *(For an I(2) model see Box 5.3, pp.93-94.)*"

[26] This estimated VAR(2) model is "var2mod" as defined in the program VAR_VECM_fxdata.prg:

```
system(model=var2mod) ;* var2mod will be later used.
variables LOGJPYUSD LOGEURUSD LOGCNYUSD
lags 1 to nlags ;* 2 ;* 19
deterministic constant
end(system)
```

(Source: VAR_VECM_fxdataPrdIII_output)

where, for example, the first row (which is for the dependent variable $\log\text{JD}_t$ in $\boldsymbol{y}_t$) of $\hat{\boldsymbol{\Phi}}_1$ (for lag 1) consists of estimated cefficients associated, respectively, with first-order lagged regressors $\log\text{JD}_{t-1}$, $\log\text{ED}_{t-1}$ and $\log\text{CD}_{t-1}$ in the estimated VAR(2) model (6).[27]

Table 9 Roots (Eigenvalues) of the Companion Matrix $\hat{\boldsymbol{\Phi}}$: $T = 761$[a]

Roots (Eigenvalues) of the Companion Matrix:		
Real	Complex/Imaginary	Modulus[b]
1.00	0.00	<u>1.00</u>
0.99	0.00	<u>0.99</u>[c]
0.98	0.00	<u>0.98</u>
-0.12	0.00	0.12
-0.04	0.00	0.04
0.00	0.00	0.00

[a]Source: VAR_VECM_fxdataPrdIII_output.

[b]The modulus is computed as described by Harris (1995, footnote 23, p.122).

[c]This and 0.98 just below might not be significantly different from unity (Harris 1995, p.89; Juselius 2006, pp.51-52). How to test it, however, is complicated (Juselius 2006, pp.51-52) and not attempted here.

Using the terminology in Harris (1995, p.89), Table 9 and Fig. 25 in Appendix A show that there are ($n \times k = 3 \times 2 =$) 6 roots of the companion matrix; ($n - r = 3 - r =$) 3 roots, which are underlined in the table, are on the unit circle (taking into account the table footnote *c*) and thus there are found ($r =$) 0 cointegration relations for Period III, which is exactly the same inference as that derived by Kojima (2019, Subsection 5.3.4) for Period V.

3.2.2 Tests on three differing nulls of lagged regressor(s) being *excluded/omitted*

Three Tests (F tests and ch-squared tests), [F], [C1] and [C2], are next conducted.A statistical note on F tests in Panel 1 of Table 10 and on

[27]The estimated constant $\hat{\mu}_1$ (which is for the dependent variable $\log\text{JD}_t$ in $\boldsymbol{y}_t$) in $\hat{\boldsymbol{\mu}}$ for the estimated model (6), for example, is 0.033. (Source: VAR_VECM_fxdataPrdIII_output)

two testing methods employed for the ch-squared tests in C1 and C2 in Panel 2 of the table is given in Kojima (2019, Subsection 5.3.3).

[F: F1, F2, F3] The block F tests, for a given equation[28] The null is that the block of lags associated with each variable (both $\log\mathrm{JD}_{t-1}$ and $\log\mathrm{JD}_{t-2}$, for example) is excluded/omitted from *a* given *equation* (an equation for dependent variable $\log\mathrm{JD}_t$, for example). See F1 through F3 in Panel 1 of Table 10.

Note, however, that "(T)he block F tests ... are not, individually, especially important. (A variable) z can, after all, still affect (another variable) x through the other equations in the (entire) system." (Doan 2007b, p.347) The following two chi-squared tests become thus more relevant and appropriate.

[C1] Chi-squared tests, for the entire model The null is that "*each variable/lag* combination ($\log\mathrm{JD}_{t-1}$, for example) is excluded/omitted from the *entire* model." See C1 in Panel 2 of Table 10, which shows that, at any conventional level of significance, every regressor except for $\log\mathrm{JD}_{t-2}$[29] and a constant is to be included in the three-exchange rate VAR model.

[C2] Global chi-squared tests, for the entire model The null is that "*all regressors* (that is, *all of the six regressors*, $\log\mathrm{JD}_{t-1}$ through $\log\mathrm{CD}_{t-2}$) across all equations are excluded/omitted with the constant being remained." See C2 in Panel 2 of Table 10: The null is easily rejected.

Business and economic implications for temporal homogeneity Combining [C1] and [C2] will lead to the inference that all of lags one and two of the three exhange rates except for $\log\mathrm{JD}_{t-2}$ are statistically and managerially important enough to explain the joint behavior of the three daily exchange rates during the sample period III: The exchange rates are statistically interrelated/interdependent via the estimated VAR(2) model, although no cointegration relationships among the three are detected (earlier in Subsection 3.2.1).

[28] Useful references include Doan (2007a, pages 158, 160), Doan (2007b, pages 345, 347) and Estima (2012, p.18).

[29] The exception of this regressor is unique to Period III (in the present paper), but was not detected for Period V in Kojima (2019).

In the context of temporal homogeneity, thus, one may argue that, for Period III as well (as Period V studied by Kojima 2019), even logCD which has been controlled carefully by the Chinese central bank and government enters into the picture as a dynamic constituent of the entire, trivariate daily exchange-rate model. Indeed, as Kojima (2019, Subsection 5.3.3) argues for Period V, one managerial implication for Period III here is that, when managerial forecasting of the three daily exchange rates is needed, they are to be considered behaving, especially over a two-day period, jointly in a (trivariate) VAR(2) manner, rather than individually or separately in a univariate time series framework. That is, singling and separating out the Chinese Yuan's exchange rate, in particular, just because of its inflexible nature does not appear appropriate for the managerial forecasting purposes.

Table 10 Tests on Three Differing Nulls of Exclusion/omission:[a] Period III; $T = 761$; Panel 1 (F tests, F1 through F3)

The Unrestricted VAR Model (5): Eq. (4) with $L = 2$		
F1:[b]		
Dependent Variable logJD		
Variable	F-Statistic	Signif
logJD	13948.451[c]	0.000
logED	0.724	0.485
logCD	0.388	0.678
F2:		
Dependent Variable logED		
Variable	F-Statistic	Signif
logJD	0.531	0.588
logED	6413.044	0.000
logCD	2.681	0.069
F3:		
Dependent Variable logCD		
Variable	F-Statistic	Signif
logJD	0.048	0.953
logED	38.486	0.000
logCD	63243.173	0.000

(Continued to Panel 2 of the Table)

[a]Source: VAR_VECM_fxdataPrdIII_output.

[b]Some remarks are in order on technical features of RATS programming (ESTIMATE instruction and LINREG instruction, in particular): The block F test results for "Dependent Variable logJD" (as generated below by ESTIMATE instruction which does NOT display degrees of freedom) can be generated, too, by LINREG instruction (which computes ordinary F statistic, Eq. (44) in Kojima 2019, Subsection 5.3.3, and does display degrees of freedom), as follows:

F test on the null of the block of two lags (both logJD{1} and logJD{2}, denoting, respectively, logJD_{t-1} and logJD_{t-2}) being excluded from the logJD equation: F(2,752)= 13948.451 with Significance Level 0.000; this is exactly the same as that generated by ESTIMATE instruction.

The same holds with the remaining F tests. F test on the null of the block of two lags (both logED{1} and logED{2}) being excluded from the logJD equation: F(2,752)= 0.724 with Significance Level 0.485. F test on the null of the block of two lags (both logCD{1} and logCD{2}) being excluded from the logJD equation: F(2,752)= 0.388 with Significance Level 0.678.

[c]The degree of freedom for the numerator of Eq. (44) in Kojima (2019, Subsection 5.3.3): $\text{df}_{num} = 2$ [the block of logJD{1} and logJD{2} being excluded]. The degree of freedom for the denominator of Eq. (44) in Kojima (2019, Subsection 5.3.3): $\text{df}_{den}(= \text{df}_{unr}) = 759(T'^{r} = T' - 2 = T - 2 = 761 - 2)$ -7(6 lagged regressors+the constant) =752. See the footnote on T, T', and T'^{r}, respectively, for the raw data, the differenced data and the residuals series for a univariate SARIMA$(p, d, q; P, D, s, Q)$ model in Kojima (2019, Subsection 3.1).

Panel 2 (Chi-squared tests, C1 and C2)[a]

C1:
Test of H_0: A System with One Regressor Excluded [A Restricted Model] against H_1: A System with All Regressors Included [An Unrestricted Model]:

A Regressor Excluded	Chi-squared	Stat Signif
logJD{1}[b]	491.230[c]	0.000[d]
logJD{2}	1.622	0.654[e]
logED{1}	515.474	0.000
logED{2}	72.180	0.000
logCD{1}	468.584	0.000
logCD{2}	12.717	0.005
Constant	4.672	0.197[f]

C2:[g]
Test of H_0: A System with No Lagged Regressors (Only with a Constant) against H_1: A System with All Regressors Included:

Regressors Excluded	Chi-squared	Stat Signif
All Lagged Regressors	11198.927[h]	0.000[i]

[a]A statistical note on two testing methods employed for the ch-squared tests in C1 and C2 here is given in Kojima (2019, Subsection 5.3.3).

[b]This denotes $\log\text{JD}_{t-1}$, a regressor at lag 1.

[c]The degree of freedom for the chi-squared statistic is: The total number of regressors, including a constant if included, in the entire *unrestricted* model (Doan 2007b, p.350) - the total number of regressors, including a constant if included, in the entire *restricted* model (Doan 2007a, p.160; 2007b, p.350) =7 regressors×3 equations - 6 regressors×3 equations=3 regressors (=the number of lagged regressors/constant, logJD{1}s, being excluded from the entire model).

[d]The null (of a system without logJD{1}, or of the two log determinants in Kojima 2019, Eq. (40) in Subsection 5.3.3, being equal) is rejected at any conventional level of significance.

[e]The null (of a system without logJD{2}, or of the two log determinants in Kojima 2019, Eq. (40) in Subsection 5.3.3, being equal) is not rejected at any conventional level of significance.

[f]The null of a constant being excluded from the entire model is not rejected at any conventional level of significance.

[g]See the remark made on T'^{τ} below Eq. (40) in Kojima (2019, Subsection 5.3.3).

[h]The degree of freedom for the chi-squared statistic is with regard to definition the same as that for C1: To be exact, 7 regressors× 3 equations - 1 regressor×3 equations=18 regressors (=6 regressors×3 equations=the number of lagged regressors being excluded from the entire model).

[i]The null (of a system without any lagged regressors, or of the two log determinants in Kojima 2019, Eq. (40) in Subsection 5.3.3, being equal) is rejected at any conventional level of significance.

4 Concluding Remarks

Studying the individual and joint behavior of three daily exchange rates (the Japanese Yen, the Euro and the Chinese Yuan), all against a U.S. dollar, during Period III (Thursday, July 21, 2005 - Thursday, July 31, 2008), this paper contrasts it with the corresponding behavior during Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016), and documents the following homogeneity in time and currency of the exchange rate behavior,[30] as summarized in Table 11.

First, for Period III, logged and then first-differenced, the Japanese Yen and the Chinese Yuan (daily rates of change in the Japanese Yen and the Chinese Yuan exchange rates) are found to behave, respectively, according to AR[4, 13, 19, 20] and AR[19], while the Euro (daily rate of change in the Euro exchange rate) a white noise.

Therefore, whereas *[a.Markov process]* in *both* Period III and Period V through 2016 the Euro exchange rate in levels obeys a random walk, *[b.non-Markov process]* for *both* periods and for *both* the Japanese Yen and the Chinese Yuan exchange rates current daily rate of change in data depends on previous daily rate of change in data *19 days in the past.* Note that the latter, *[b]*, is observed even while during both two periods Japan employs a flexible exchange rate system and China a managed flexible exchange rate system.

Second, the temporal and cross-currency homogeneity observed for the two periods (even with the intervention model for the Chinese Yuan exchange rate for Period V through 2016) may not be mere coincidence as to time and currency and could be more than statistical in nature. What essentially or deductively lies behind the economic implications such as Markov versus non-Markov (including the statistically significant ϕ_{19} in common) is not readily evident, however. Further investigation is needed in qualitative business and economic dimensions: This will be one topic to be studied in the future work, two statistical preliminaries to which are detailed in Subsection 2.2.4.

Third, noting that Period III turns out the third longest period of time (in Table 1) when the Yuan was continuously less managed/controlled by the central bank in China under (managed) flexible exchange rate system, the unrestricted VAR modeling for the period, whose lag length turns out two (days) (exactly the same as for Period V), detects no cointegration relationships among the three daily exchange rates, and yet the

[30] For what is meant in the present paper by "homogeneity" as such see Section 1.

chi-squared tests for their unrestricted VAR model (that is, VAR model with no a priori restrictions/cointegrations) show that even the China's Yuan exchange rate which has been controlled carefully by the Chinese central bank and government enters into the picture as a statistically significant constituent of the trivariate daily exchange-rate model.

Table 11 Summary Table: Homogeneity in Time and Currency of the Exchange Rate Behavior

		Present Paper:	Kojima (2019):
		Time Periods Contrasted	
A. Logged Daily Exchange Rates in First Differences (Daily Rates of Change in Exchange Rates):[a]			
Dimension	Currency	Period III[b]	Period V[c] through 2016[d]
Univariate	Yen	AR[4, 13, 19, 20][e]	AR[19][f]
	Euro	white noise[g]	white noise
	(Euro in *Levels*	*random walk*[h]	*random walk*)
	Yuan	AR[19][i]	Intervention Model with ϕ_{19}[j]
B. Logged Daily Exchange Rates in Levels:			
Dimension	Currency	Period III	Period V
Trivariate	Yen, Euro and Yuan	Unrestricted VAR(2) model:[k] 1. No cointegration relationships are detected;[l] and 2. The three are still statistically interdependent via the estimated VAR(2) model (6).[m]	

[a]For logged daily JD and CD in *second* differences and their nonstationarity, see Subsection 2.2.4.

[b]See Panel 1 of Table 1.

[c]See Panel 2 of Table 1.

[d]Period V and Tuesday, August 11, 2015 - Friday, December 30, 2016 combined.

[e]See Table 4.

[f]See Table 5.

[g]See Table 6.

[h]The italic models are a *Markov process*. The remaining, non-italic univariate models are all a *non-Markov process*.

[i]See Table 7.

[j]See Table 8.

[k]VAR Model (5) involving up to 2 lags for both periods, III and V, with no a priori restrictions or cointegrations. See Subsections 3.1 and 3.2.

[l]See Subsection 3.2.1.

[m]See Subsection 3.2.2; the parameter estimates of VAR(2) model are arrayed in the companion matrix $\hat{\mathbf{\Phi}}$ (including $\hat{\mathbf{\Phi}}_1$ for lag 1 and $\hat{\mathbf{\Phi}}_2$ for lag 2) in Subsection 3.2.1.

Thus, for Period III, too, as concluded for Period V by Kojima (2019), singling and separating out the Yuan's exchange rate, in particular, because of its inflexible nature does not appear appropriate, although no cointegration relationships exist either a priori or a posteriori among the three daily exchange rates. The VAR modeling of the three may be still meaningful for the managerial forecasting purposes for Period III as well.

As may be seen from Table 1, another future study will be univariate time series analysis of the three exchange rates for Period VI (Tuesday, September 1, 2015 - Friday, December 20, 2019/Present) as well as Period I (Monday, January 3, 1994 - Tuesday, December 31, 1996). As shown and explained in Kojima (2019, Fig. 3), the Yuan's exchange rate in levels appears to have an additive outlier on Monday, December 19, 1994, implying an intervention model (Kojima 2019, Eq. (16) in Subsection 4.3.1) to be identified and estimated for Period I (as for Period V through 2016).

Meanwhile, the behavior of the daily Yuan falling and firming during Period VI is seen to involve no such outliers and, especially since 2018, is most likely associated with the U.S.-China trade war (see Fig. 5).

One particular question of interest for the future work for Period VI may then be whether the AR parameter ϕ_{19} will again play such a statistical and/or business and economic role for both Yen and Yuan exchange rates as documented for Period III and Period V through 2016, respectively, by the present paper and Kojima (2019). With time being thus expanded to include Periods I and VI as well, the question will be again a research topic of homogeneity in time and currency.

Appendices

The table just below lists the source of each figure and table, which is available from the author upon request:

Figure Number	Source
1	Published in 2019: MacRATS: BJidentify_fxdata.prg
2 - 4	Published in 2020: MacRATS: BJidentify_fixdata_Jul212005-Jul312008.prg
Figure Appendix:	
12, 15 and 19	Published in 2020: MacRATS: BJidentify_fixdata_Jul2005-Jul2008.prg
13, 16 and 20	Published in 2020: MacRATS: BJidentify_fixdata_Jun2010-Dec2016.prg
Table Number	**Source**
1	BJidentify_fixdata_Jan31994-Dec311996output, BJidentify_fixdata_Jul212005-Jul312008output and BJidentify_fxdataoutput
3	BJestimate_outputJPYUSD1
4	BJestimate_outputJPYUSD2
5	BJetimate_jpyModelBoutput
6	BJestimate_outputEURUSD
7	BJestimate_outputCNYUSD
8	BJetimate_cnyModelB-ARMA_IntrvModeloutput
9 and 10	VAR_VECM_fxdataPrdIII_output
Table Appendix:	
12	VARLAG_fxdataPrdIII_output

A Figure Appendix

This appendix contains figures drawn for (i) univariate time series analysis of the the exchange rates and (ii) roots of the companion matrix.

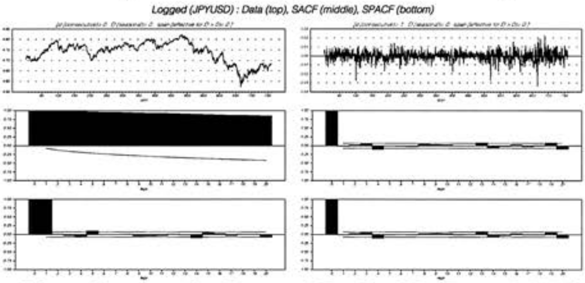

Figure 10 Identification for Logged Daily JD, Period III (Thursday, July 21, 2005 - Thursday, July 31, 2008): Levels and First Differences.

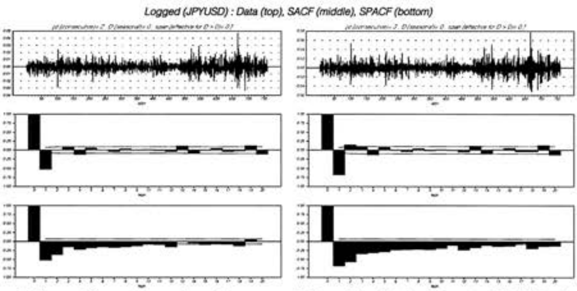

Figure 11 Identification for Logged Daily JD, Period III: Second- and Third-order Differences.

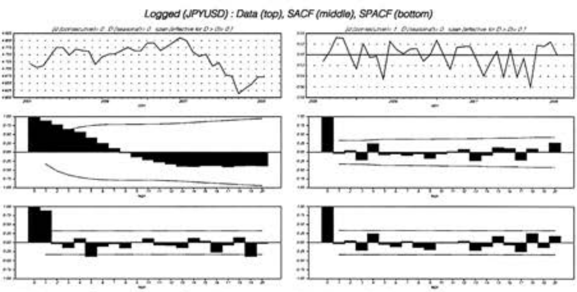

Figure 12 Identification for Logged Monthly JD, Period III (July 2005 - July 2008): Levels and First Differences.

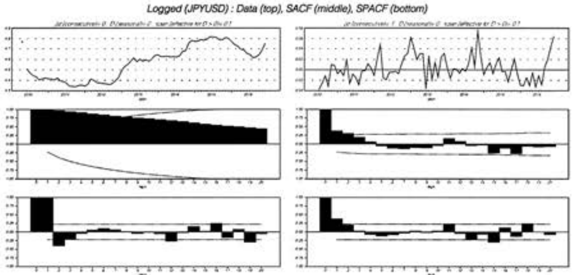

Figure 13 Identification for Logged Monthly JD, Period V through 2016 (June 2010 - December 2016): Levels and First Differences.

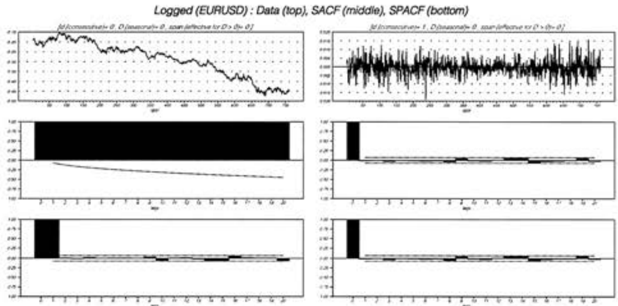

Figure 14 Identification for Logged Daily ED, Period III: Levels and First Differences.

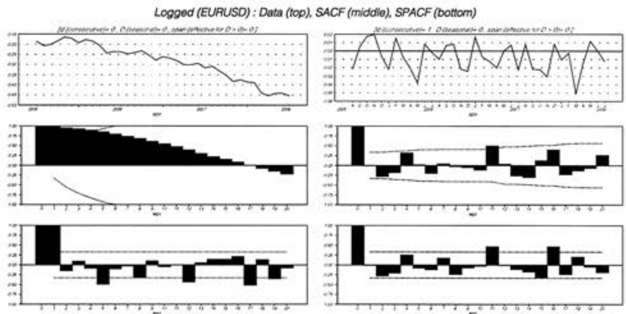

Figure 15 Identification for Logged Monthly ED, Period III: Levels and First Differences.

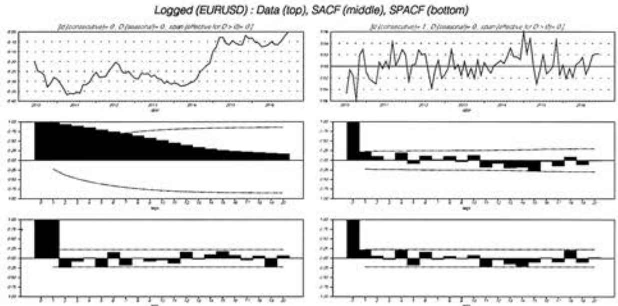

Figure 16 Identification for Logged Monthly ED, Period V through 2016: Levels and First Differences.

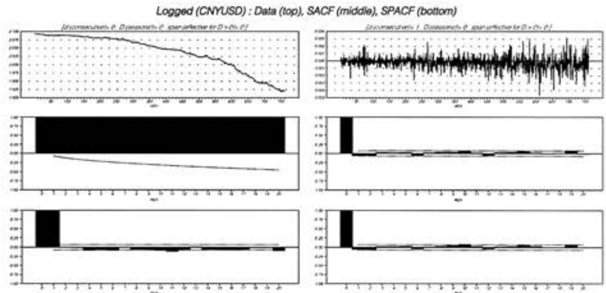

Figure 17 Identification for Logged Daily CD, Period III: Levels and First Differences.

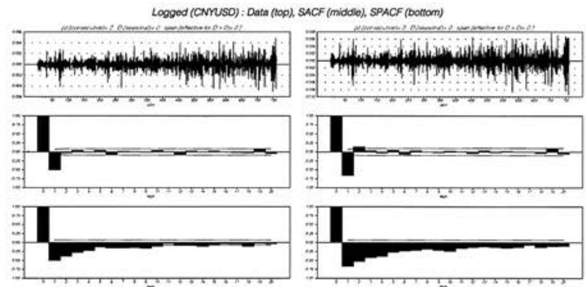

Figure 18 Identification for Logged Daily CD, Period III: Second- and Third-order Differences.

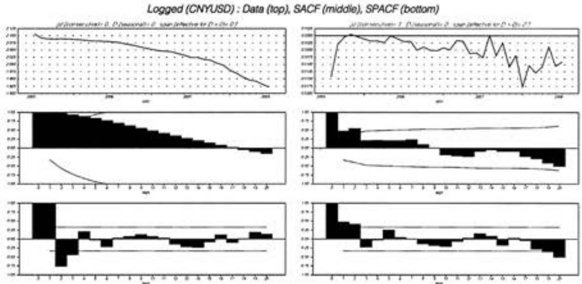

Figure 19 Identification for Logged Monthly CD, Period III: Levels and First Differences.

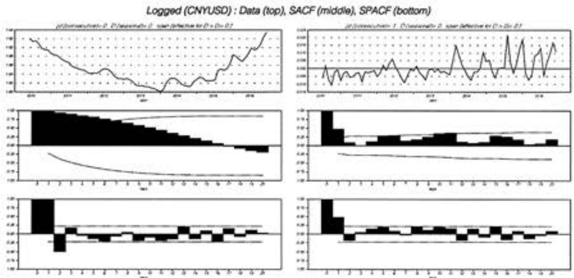

Figure 20 Identification for Logged Monthly CD, Period V through 2016: Levels and First Differences.

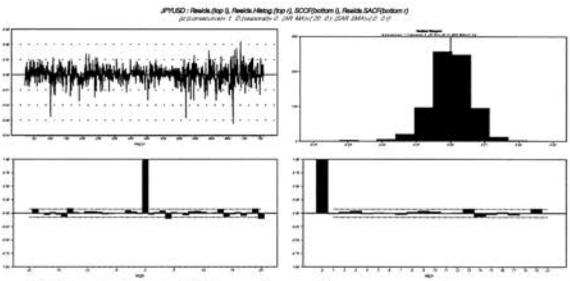

Figure 21 AR[4, 20] Model with a Constant: Estimation for Logged Daily JD in First Differences (Daily Rate of Change in JD), Period III.

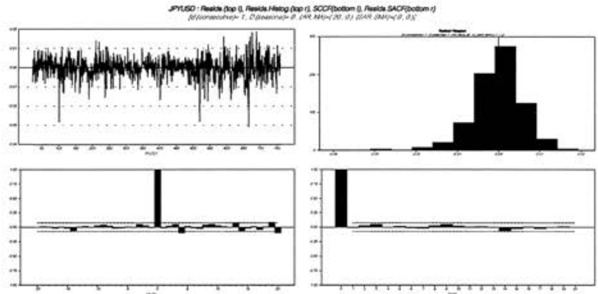

Figure 22 AR[4, 13, 19, 20] Model without a Constant: Estimation for Logged Daily JD in First Differences (Daily Rate of Change in JD), Period III.

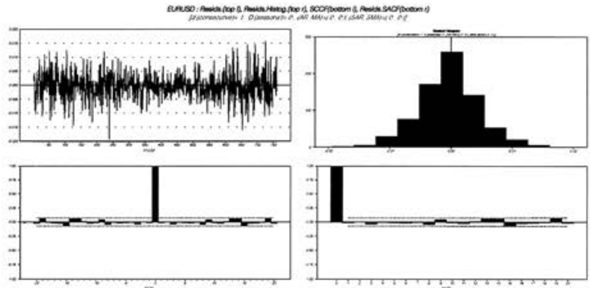

Figure 23 White Noise Model without a Constant: Estimation for Logged Daily ED in First Differences (Daily Rate of Change in ED), Period III.

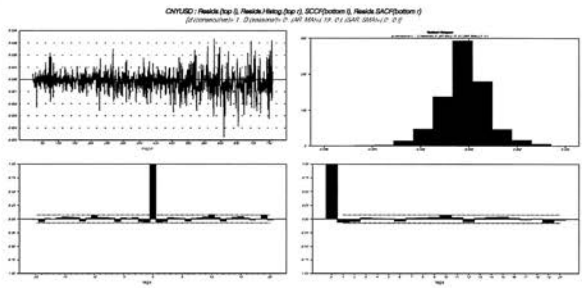

Figure 24 AR[19] Model without a Constant: Estimation for Logged Daily CD in First Differences (Daily Rate of Change in CD), Period III.

B Table Appendices

B.1 For footnote d to Panel 1 of Table 1

Exact dates of "Skipped/Missing" for Period I in Table 1 for CD are as follows: Mon., Jan.17, 1994, Mon., Feb.21, 1994, Fri., Apr.1, 1994,

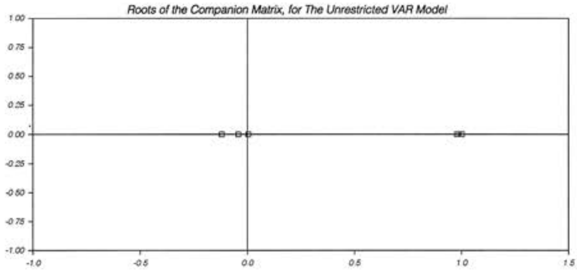

Figure 25 Roots (Eigenvalues) of the Companion Matrix for Logged Daily Exchange Rates, Period III.

Mon., Apr.4, 1994, Mon., May 30, 1994, Mon. Jul.4, 1994, Thu., Nov.24, 1994, Mon., Jan.16, 1995, Mon., Feb.20, 1995, Mon., May 29, 1995, Tue., Jul.4, 1995. (See fxdata_Jan1994-Dec2016_MacRATS.xls, which is available from the author upon request.)

B.2 For footnote c to Panel 2 of Table 1

grCD for Period V has one missing which is not observed either for Period I or III in Panel 1 of Table 1, simply due to the technical programming reason: For Period V, statistics(fractiles) grCNYUSD 1 1286 which does specify the sample period for grCNYUSD (see the third source in the table), whereas for Periods I and III statistics(fractiles) i which does not specify the sample period for dofor i = grJPYUSD grEURUSD grCNYUSD (see the first and second sources). In other words, for Period V statistics(fractiles) grCNYUSD 2 1286 would have simply led to no missing.

B.3 For Lag Length

This appendix tabulates test results for setting the lag length of the unrestricted VAR model.

Table 12 Setting the Lag Length for VAR:[a] Period III; $T = 761$; Panel 1 ([A], [B])

[A] H_1: longernlags= 2, H_0: shorternlags= 1

a. Using RATIO:
Log Determinants are -35.219 -35.101
Chi-Squared(9)[b]= 88.792 with Significance Level 0.000

b. Using calculated statistic:
Chi-Squared(9)= 86.703 with Significance Level 0.000

c. Using VARLagSelect procedure:

Lags	AICC			
0	-8960.267			
1	-20155.676			
2	-20227.015*			
Lags	AIC	SBC	LR Test	P-Value
1	-26.558	-26.485		
2	-26.650*	-26.522*	60.920	0.000

[B] H_1: longernlags= 3, H_0: shorternlags= 2

a. Using RATIO:
Log Determinants are -35.238 -35.215
Chi-Squared(9)[c]= 16.973 with Significance Level 0.049

b. Using calculated statistic:
Chi-Squared(9)= 14.262 with Significance Level 0.113

c. Using VARLagSelect procedure:

Lags	AICC			
0	-8949.599			
1	-20126.438			
2	-20197.565*			
3	-20196.343			
Lags	AIC	SBC	LR Test	P-Value
1	-26.558	-26.485		
2	-26.650*	-26.522*	60.920	0.000
3	-26.644	-26.462	-12.253	NA

(Continued to Panel 2 of the Table)

[a]Doan (2007b, pp.348-349) gives an example of testing a lag length, whose programming is applied in the present table. Source: VARLAG_fxdataPrdIII_output.

[b]The degree of freedom 9 = the total number of (lagged) regressors under the alternative H_1−that under the null $H_0 = 2 \times 3 \times 3 - 1 \times 3 \times 3 = 18 - 9 =$ the number of parameters *excluded* in H_0 as against H_1.

[c]The degree of freedom 9 = the total number of (lagged) regressors under the alternative H_1−that under the null $H_0 = 3 \times 3 \times 3 - 2 \times 3 \times 3 = 27 - 18 =$ the number of parameters *excluded* in H_0 as against H_1.

Panel 2 ([C], [D])

[C] H_1: longernlags= 4, H_0: shorternlags= 2

a. Using RATIO:

Log Determinants are -35.248 -35.213

Chi-Squared(18)[a]= 26.129 with Significance Level 0.097

b. Using calculated statistic:

Chi-Squared(18)= 21.773 with Significance Level 0.242

c. Using VARLagSelect procedure:

Lags	AICC
0	-8938.654
1	-20098.157
2	-20169.192*
3	-20167.977
4	-20158.769

Lags	AIC	SBC	LR Test	P-Value
1	-26.558	-26.485		
2	-26.650*	-26.522*	60.920	0.000
3	-26.644	-26.4621	-12.253	NA
4	-26.630	-26.393	-19.005	NA

[D] H_1:longernlags= 20,[b] H_0: shorternlags= 2

a. Using RATIO:

Log Determinants are -35.486 -35.197

Chi-Squared(162)[c]= 196.393 with Significance Level 0.034

b. Using calculated statistic:

Chi-Squared(162)= 181.625 with Significance Level 0.139

c. Using VARLagSelect procedure:

Lags	AICC
0	-8778.346
1	-19660.125
2	-19730.509*
3	-19729.503
4	-19722.248
5	-19721.708
6	-19708.740
7	-19706.962
8	-19695.950
9	-19690.996
10	-19677.626
11	-19663.192
12	-19654.756
13	-19646.354
14	-19636.385
15	-19640.942
16	-19634.526
17	-19627.270
18	-19609.844
19	-19600.280
20	-19587.617

Lags	AIC	SBC	LR Test	P-Value
1	-26.558	-26.485		
2	-26.650*	-26.522*	60.920	0.0000
3	-26.644	-26.462	-12.253	NA
4	-26.630	-26.393	-19.005	NA
5	-26.627	-26.336	-10.264	NA
6	-26.610	-26.265	-20.213	NA
7	-26.604	-26.205	-12.026	NA
8	-26.590	-26.1367	-17.887	NA
9	-26.592	-26.084	-6.339	NA
10	-26.572	-26.010	-21.885	NA
11	-26.552	-25.936	-22.179	NA
12	-26.539	-25.890	-16.373	NA
13	-26.523	-25.801	-18.021	NA
14	-26.513	-25.737	-14.016	NA
15	-26.518	-25.689	-2.192	NA
16	-26.505	-25.623	-15.493	NA
17	-26.494	-25.558	-14.386	NA
18	-26.466	-25.478	-26.353	NA
19	-26.449	-25.408	-17.494	NA
20	-26.434	-25.341	-16.402	NA

[a] The degree of freedom 18 = the total number of (lagged) regressors under the alternative H_1 −that under the null $H_0 = 4 \times 3 \times 3 - 2 \times 3 \times 3 = 36 - 18$ = the number of parameters *excluded* in H_0 as against H_1.

[b] For why as many as 20 days here see Kojima (2019, Subsection 5.3.1).

[c] The degree of freedom 162 = the total number of (lagged) regressors under the alternative H_1 −that under the null $H_0 = 20 \times 3 \times 3 - 2 \times 3 \times 3 = 180 - 18$ = the number of parameters *excluded* in H_0 as against H_1.

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